QCD phase diagram at large N_c

The standard lore:

QCD Phase Diagram vs temperature, T, and quark chemical potential, μ

One transition, chiral = deconfined, "semicircle"

Large N_c:

Two transitions, chiral $\neq$ deconfinement

Not just a critical end point, but a new "*quarkyonic*" phase:

Confined, chirally symmetric baryons: *massive*, parity doubled.

Work exclusively in rotating arm approximation...

McLerran & RDP, 0706.2191, to appear in NPA.

The first semicircle

Cabibbo and Parisi ‘75: Exponential (Hagedorn) spectrum limiting temperature, *or* transition to new, “unconfined” phase. One transition.

Punchline today: below for chiral transition, deconfinement splits off at finite μ.

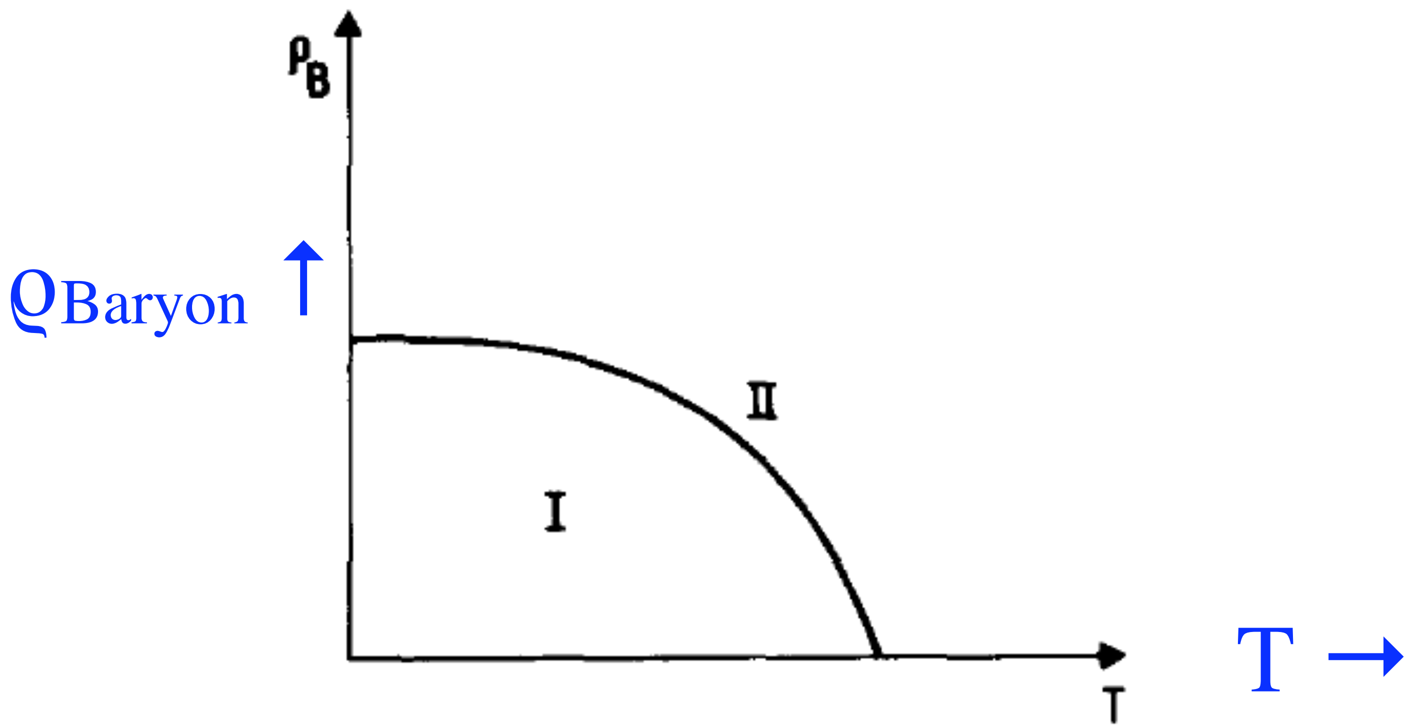
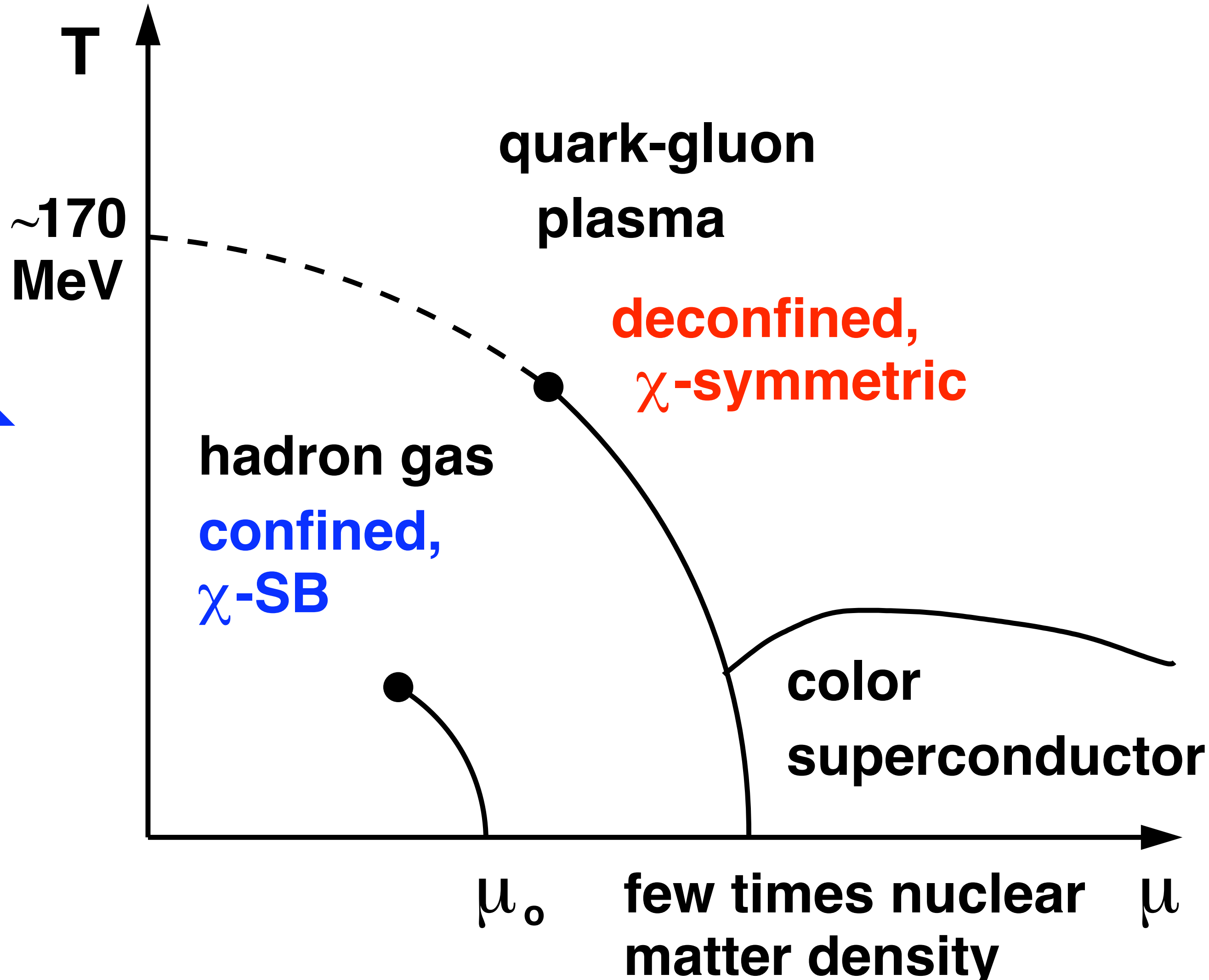


Fig. 1. Schematic phase diagram of hadronic matter. ρ_B is the density of baryonic number. Quarks are confined in phase I and unconfined in phase II.

Phase diagram, ~ ‘06

Lattice, $T \neq 0$, $\mu = 0$: two possible transitions, occur at same T. Karsch ’06

Persists at $\mu \neq 0$? Stephanov, Rajagopal, & Shuryak ‘98:
Critical end point where crossover becomes 1st order trans.?



$T \uparrow$ $\mu \rightarrow$

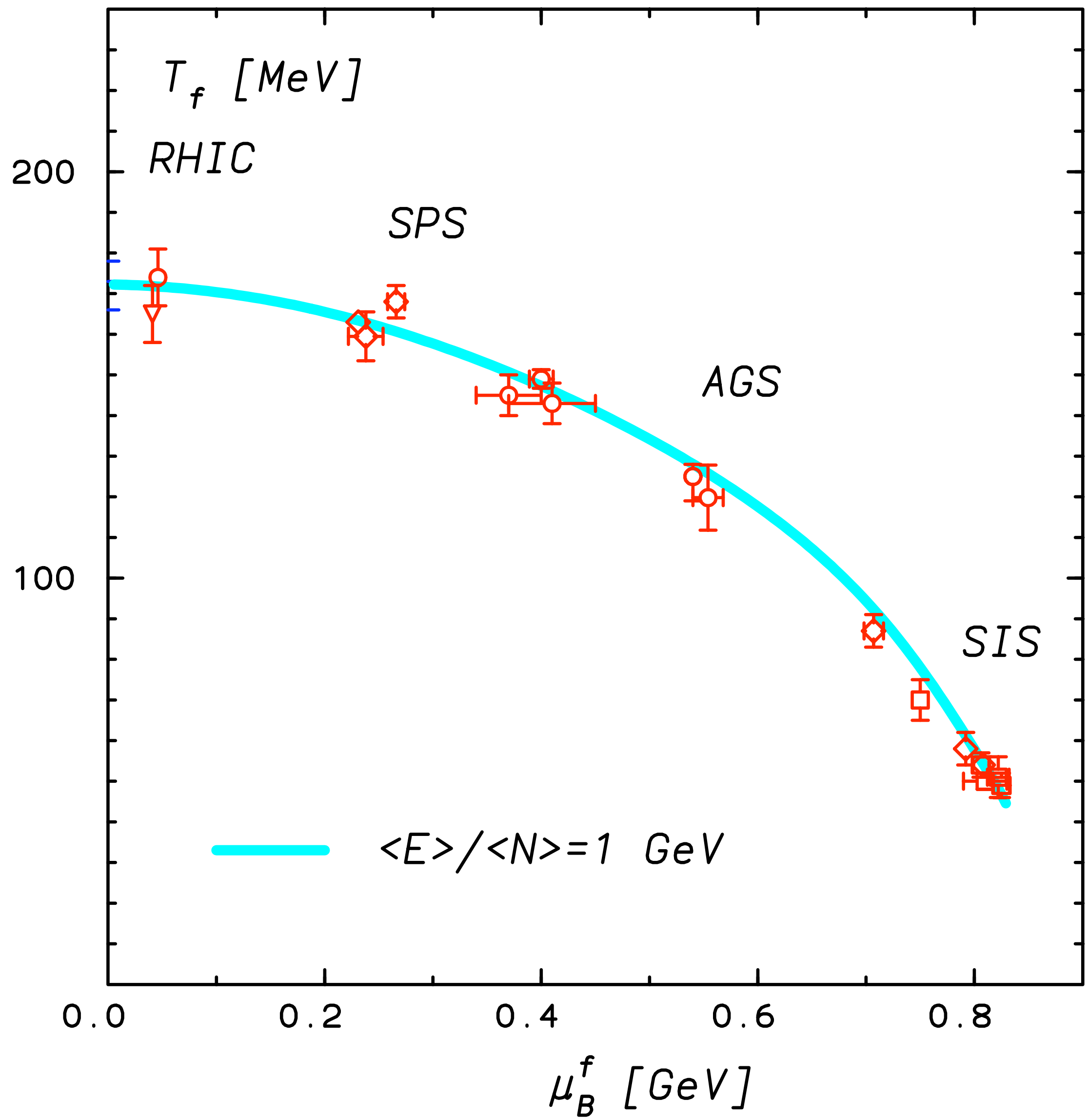
Experiment: freezeout line

Cleymans & Redlich '99: Line for chemical equilibriation at freezeout

~ semicircle.

N.B.: for T = 0, goes down to ~ nucleon mass.

T ↑

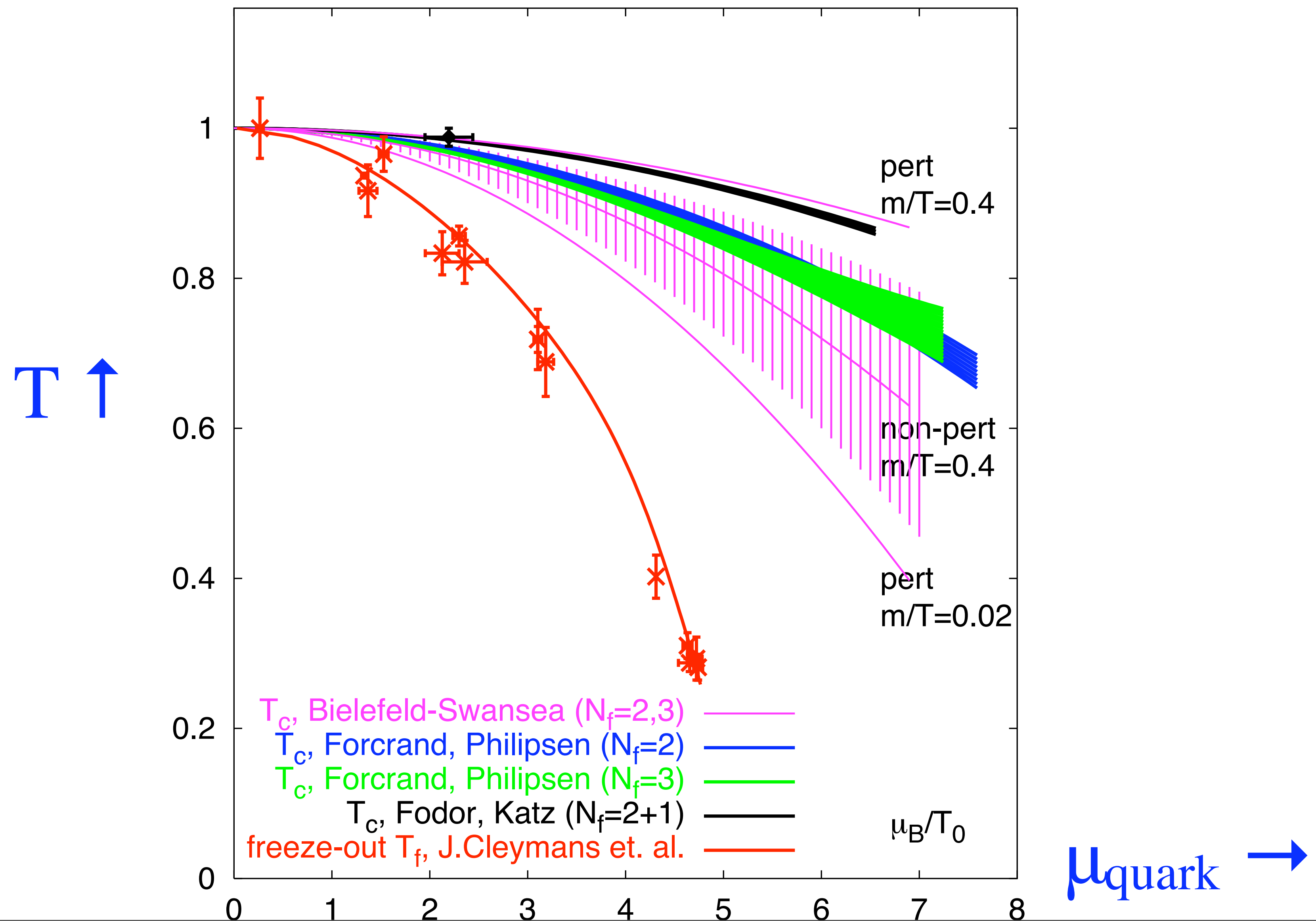


μ_{Baryon} →

Experiment vs. Lattice

Lattice "transition" appears *above* freezeout line? Schmidt '07

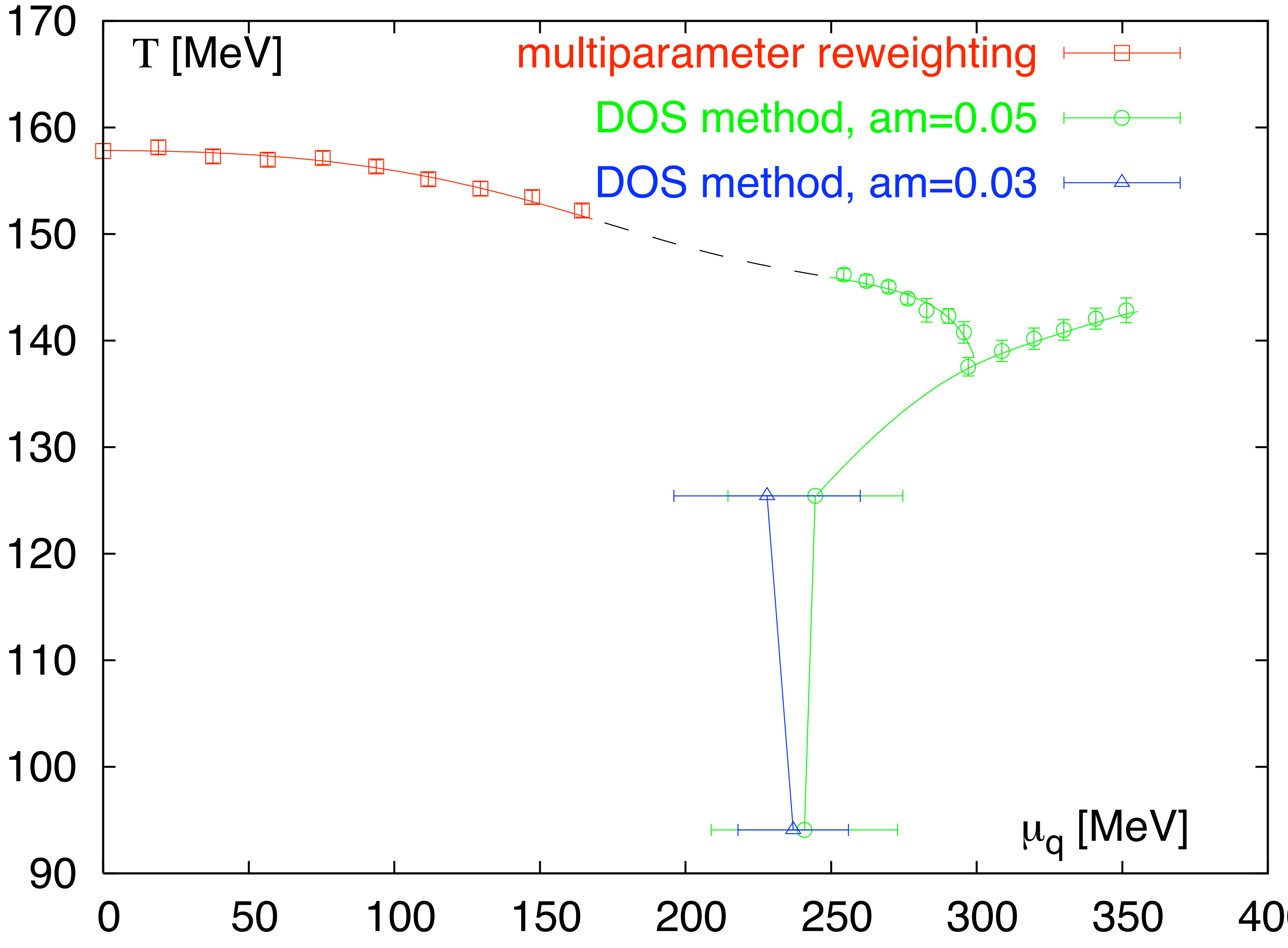
N.B.: small change in T_c with μ?



Lattice T_c , vs μ

Rather small change in T_c vs μ? Depends where μ_c is at $T = 0$. Fodor & Katz '06

$T \uparrow$



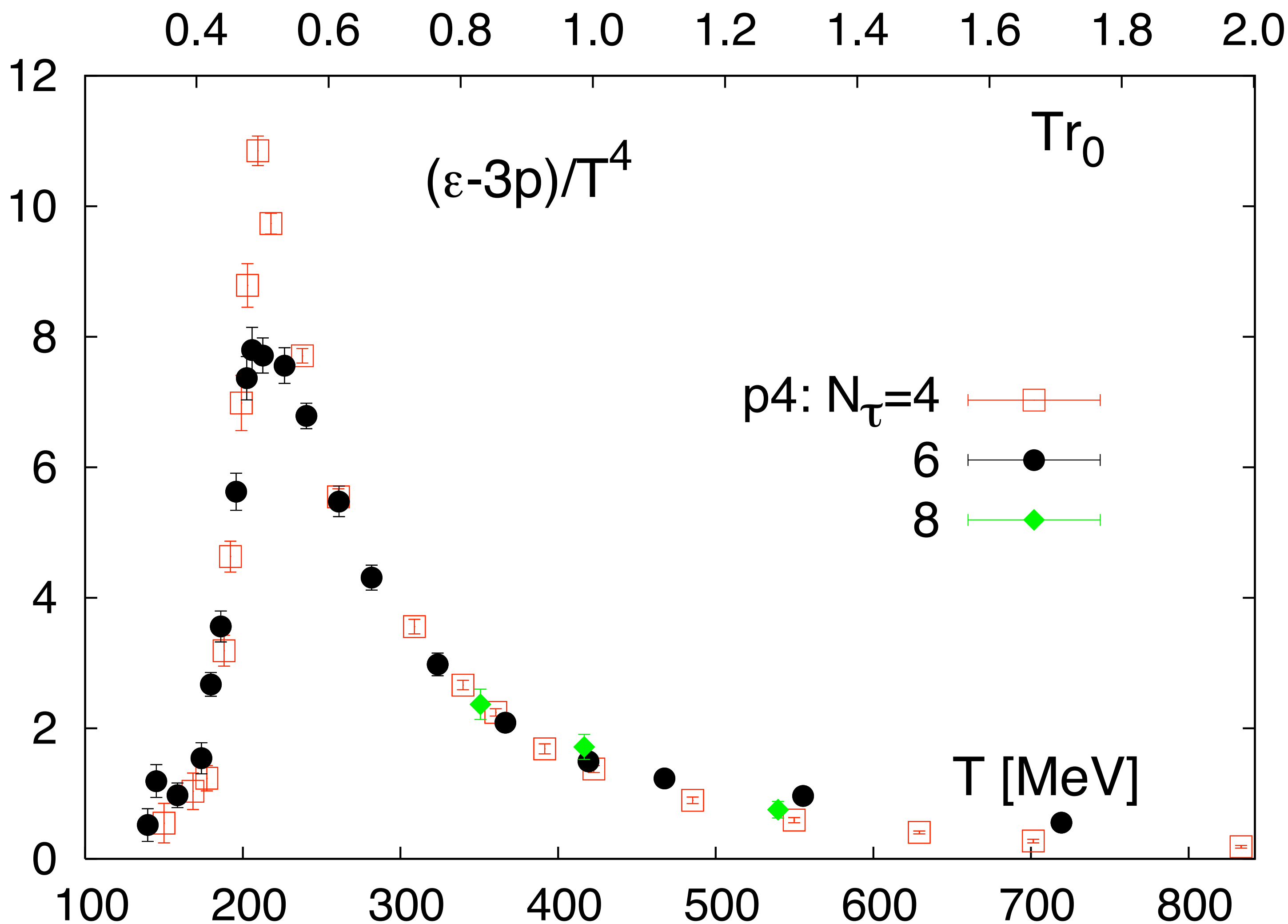
$\mu_{quark} \rightarrow$

Lattice pressure

For all μ, pressure fits well with (Cheng et al., 0710.0354)

$$p(T) = f_{pert}\, T^4 - B_{fuzzy}\, T^2 - B_{MIT} + \ldots$$

$(e\text{-}3p)/T^4$ ↑



T →

EoS of nuclear matter

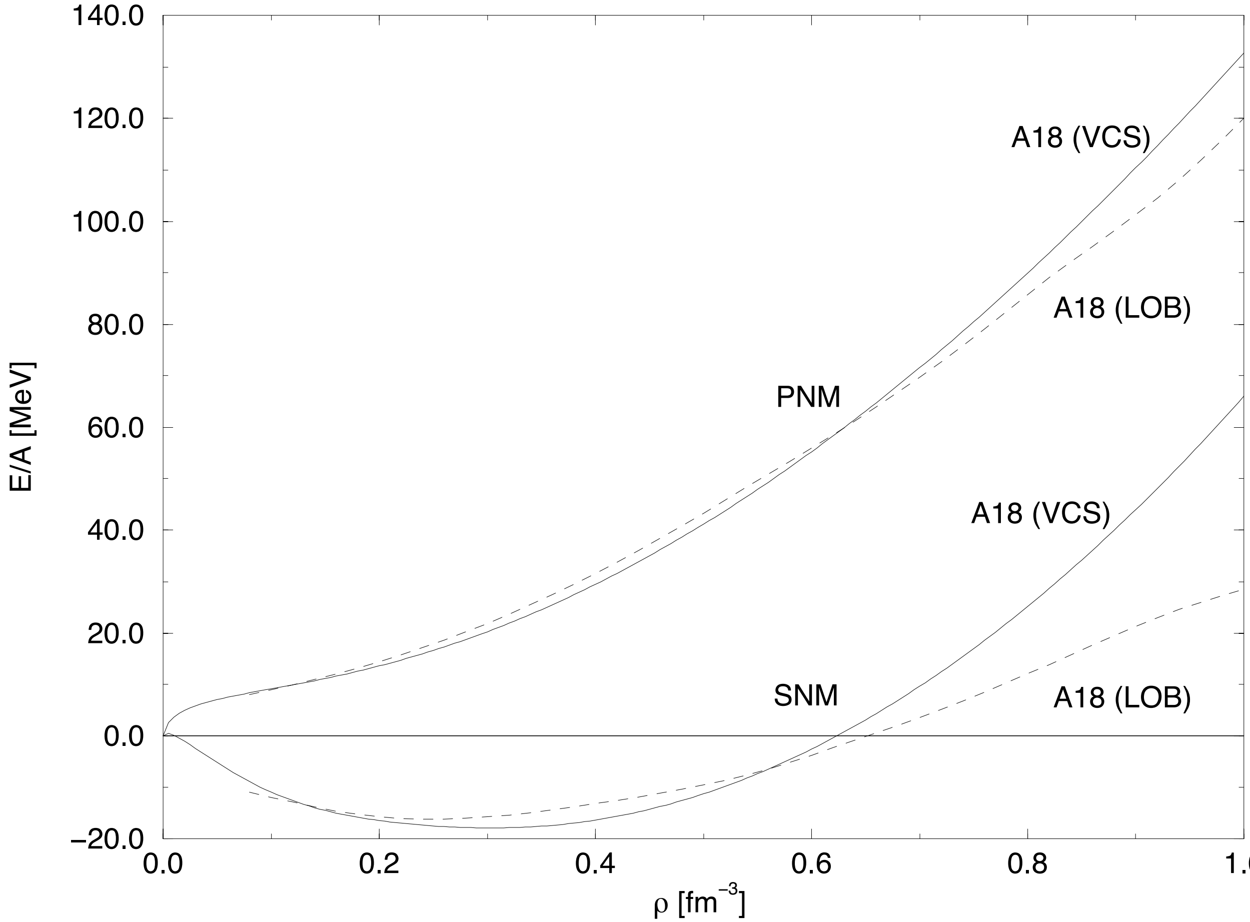
Akmal, Panharipande, & Ravenhall '98: Equation of State for nuclear matter, T=0

E/A = energy/nucleon. Fits to various nuclear potentials

Anomalously small: binding energy of nuclear matter 15 MeV!

Calc's reliable to ~ twice nuclear matter density.

E/A ↑



ρBaryon →

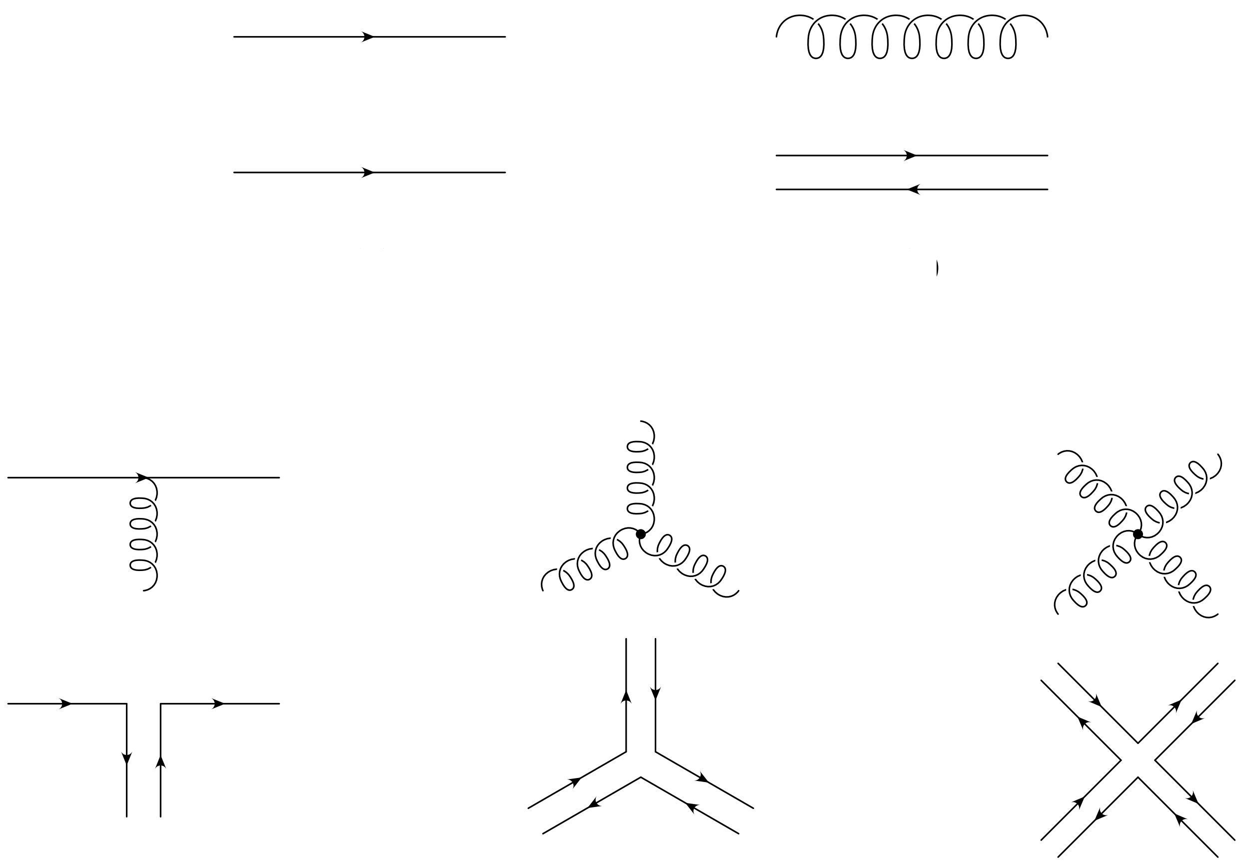
Expansion in large N_c

‘t Hooft ’74: let $N_c \to \infty$, with $\lambda = g^2 N_c$ fixed.

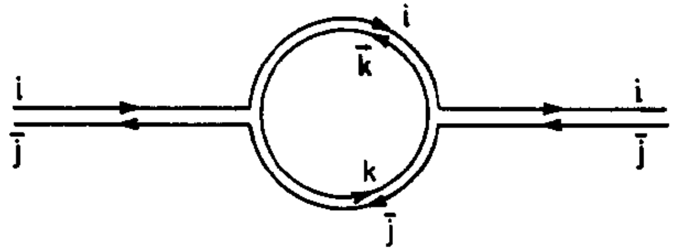
$\sim N_c^2$ gluons in adjoint representation, vs $\sim N_c$ quarks in fundamental rep. $\Rightarrow$

Large N_c dominated by *gluons* (iff N_f = # quark flavors *small*)

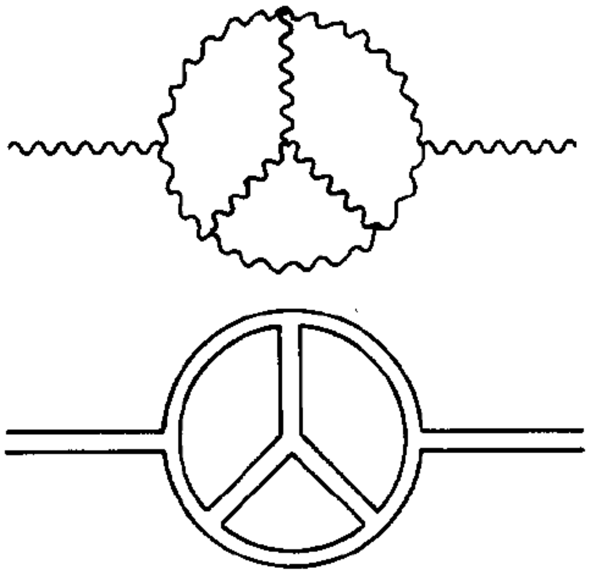
“Double line” notation. Useful at small N_c (Yoshimasa Hidaka & RDP)

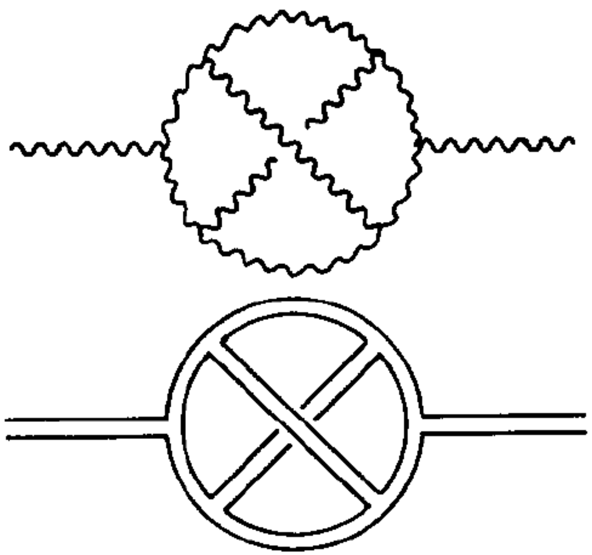


Large N_c : “planar” diagrams



$\sim g^2 N_c = \lambda$

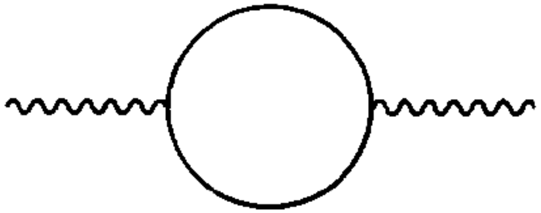


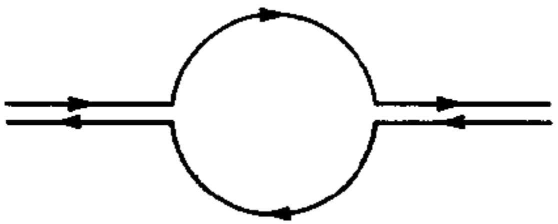


Planar diagram, $\sim \lambda^2$

Non-planar diagram, $\sim \lambda^2 / N_c$
Suppressed by $1/N_c$

Quark loops suppressed at large N_c





$$\sim g^2 = \lambda \times \frac{1}{N_c}$$

Quark loops are suppressed at large N_c, *only* if N_f (= # quark flavors) is held fixed

Thus: limit of large N_c , small N_f

Quarks can be introduced as external sources.

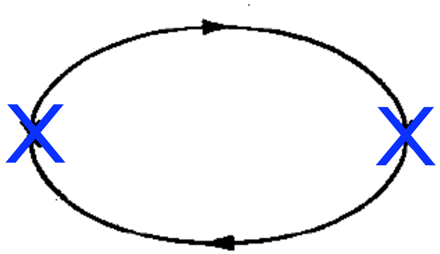
Analogous to "quenched" approximation, expansion about $N_f = 0$.

Veneziano '78: take both N_c and N_f large. Not well understood.

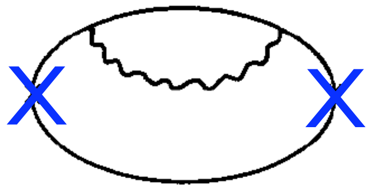
Form factors at large N_c

J ~ (gauge invariant) mesonic current

$$< J(x)J(0) > \sim N_c$$



Infinite # of planar diagrams for $< J\, J >$:



Confinement => sum over mesons, form factors ~ $N_c^{1/2}$

$$< J(x)J(0) > \sim \int d^4p\, e^{ip\cdot x} \sum_n < 0|J|n > \frac{1}{p^2 + m_n^2} < n|J|0 >$$

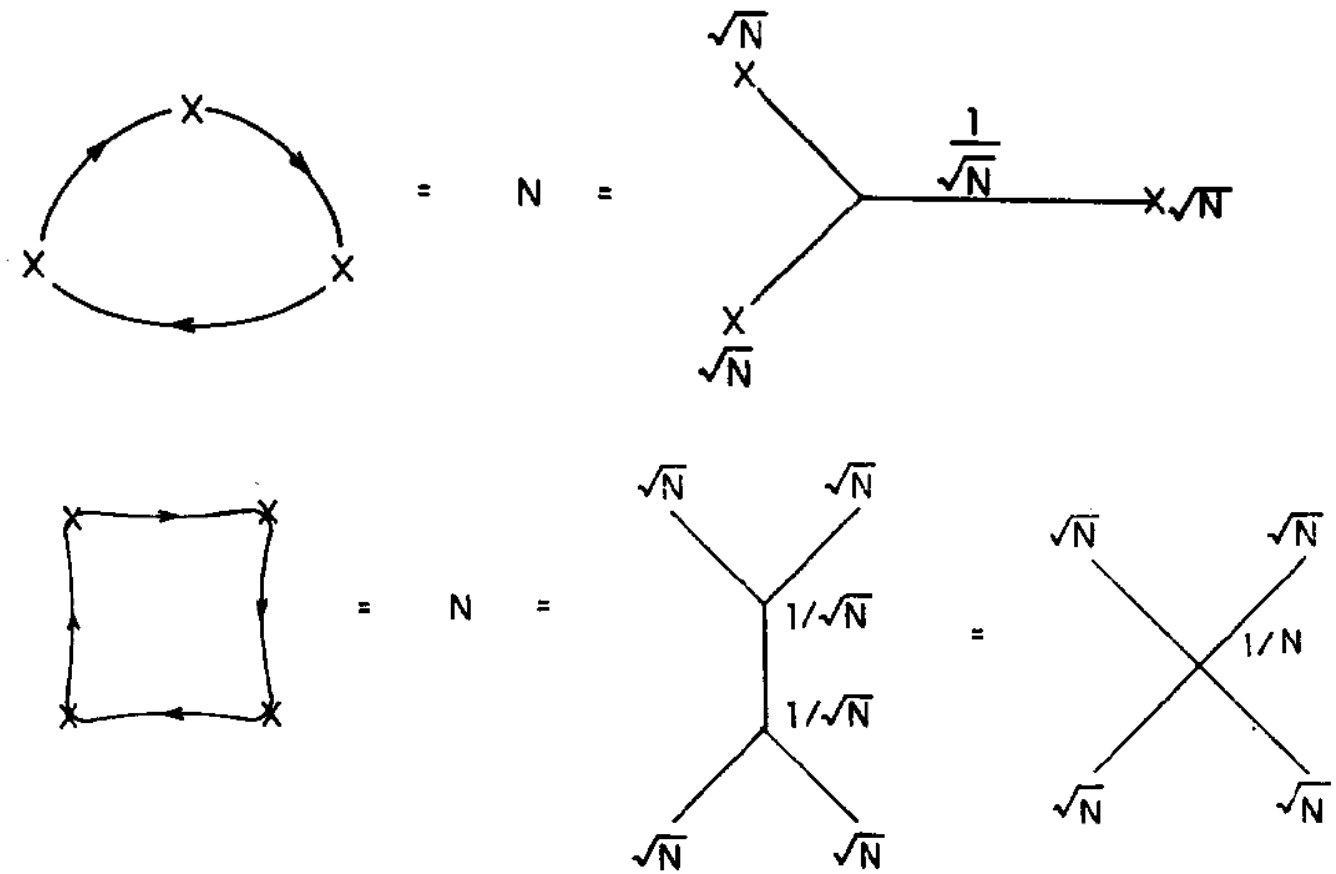
$$< J(x)J(0) > \sim \; N_c \quad \Rightarrow \; < 0|J|n > \sim \sqrt{N_c} \;\text{ if }\; m_n \sim 1$$

Mesons & glueballs *free* at $N_c = \infty$

With form factors ~ $N_c^{1/2}$, 3-meson couplings ~ $1/N_c^{1/2}$; 4-meson, ~ $1/N_c$
For glueballs, 3-glueball couplings ~ $1/N_c$, 4-glueball ~ $1/N_c^2$

Mesons and glueballs don't interact at $N_c = \infty$.

Large N limit *always* (some) classical mechanics Yaffe '82

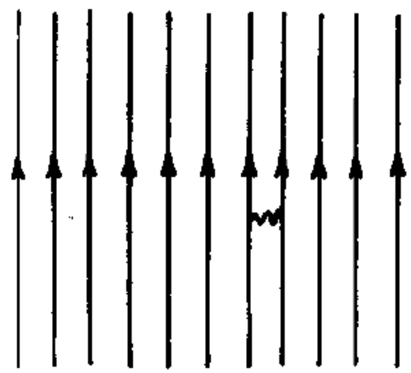


Baryons at large N_c

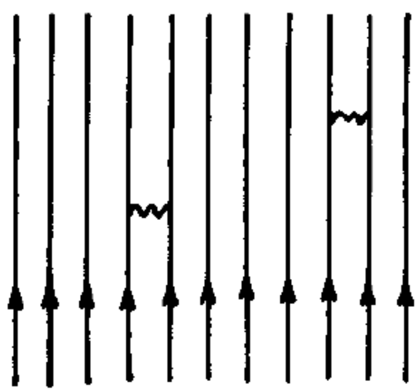
Witten '79: Baryons have N_c quarks, so nucleon mass $M_N \sim N_c \Lambda_{QCD}$.

Baryons like "solitons" of large N_c limit (~ Skyrmion)

Leading correction to baryon mass:



$$g^2 \times N_c \times N_c \sim \lambda N_c$$

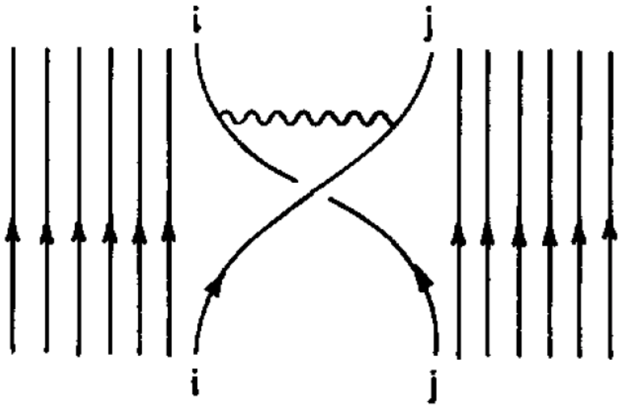


Appears $\sim g^4 N_c^4 \sim \lambda^2 N_c^2$?

No, iteration of average potential, mass still $\sim N_c$.

Baryons are *not* free at $N_c = \infty$

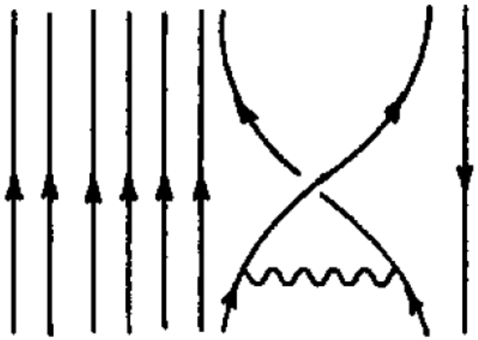
Baryons interact strongly. Two baryon scattering ~ N_c :



$$g^2 \times N_c \times N_c \sim \lambda\, N_c$$

Scattering of three, four... baryons also ~ N_c

Mesons also interact strongly with baryons, ~ N_c^0 ~ 1



$$g^2 \times N_c \sim \lambda$$

Skyrmions and $N_c = \infty$ baryons

Witten '83; Adkins, Nappi, Witten '83: Skyrme model for baryons

$$\mathcal{L} = f_\pi^2 \, \mathrm{tr}|V_\mu|^2 + \kappa \, \mathrm{tr}[V_\mu, V_\nu]^2 \ , \ V_\mu = U^\dagger \partial_\mu U \ , \ U = e^{i\pi/f_\pi}$$

Baryon soliton of pion Lagrangian: $f_\pi \sim N_c^{1/2}$, $\kappa \sim N_c$, mass $\sim f_\pi^2 \sim \kappa \sim N_c$.

Single baryon: at $r = \infty$, $\pi^a = 0$, $U = 1$. At $r = 0$, $\pi^a = \pi \, r^a/r$.
Baryon number topological: Wess-Zumino '71, Witten '83.

Dashen & Manohar '93, Dashen, Jenkins, & Manohar '94:
Huge degeneracy of baryons: multiplets of isospin and spin, $I = J$: $1/2 \ldots N_c/2$.
Obvious in Skyrme model, as collective coordinates of soliton.

Baryon-meson coupling $\sim N_c^{1/2}$, cancellations from extended $SU(2\,N_f)$ symmetry.

Towards the phase diagram at $N_c = \infty$

As example, consider gluon polarization tensor at zero momentum.
(~ Debye mass^2 at leading order, gauge invariant)

$$\Pi^{\mu\mu}(0) = g^2 \left(\left(N_c + \frac{N_f}{2} \right) \frac{T^2}{3} + \frac{N_f \mu^2}{2\pi^2} \right) = \lambda \, \frac{T^2}{3} \; , \; N_c = \infty$$

For $\mu \sim N_c^0 \sim 1$, at $N_c = \infty$ the gluons are blind to quarks.

When $\mu \sim 1$, deconfining transition temperature $T_d(\mu) = T_d(0)$

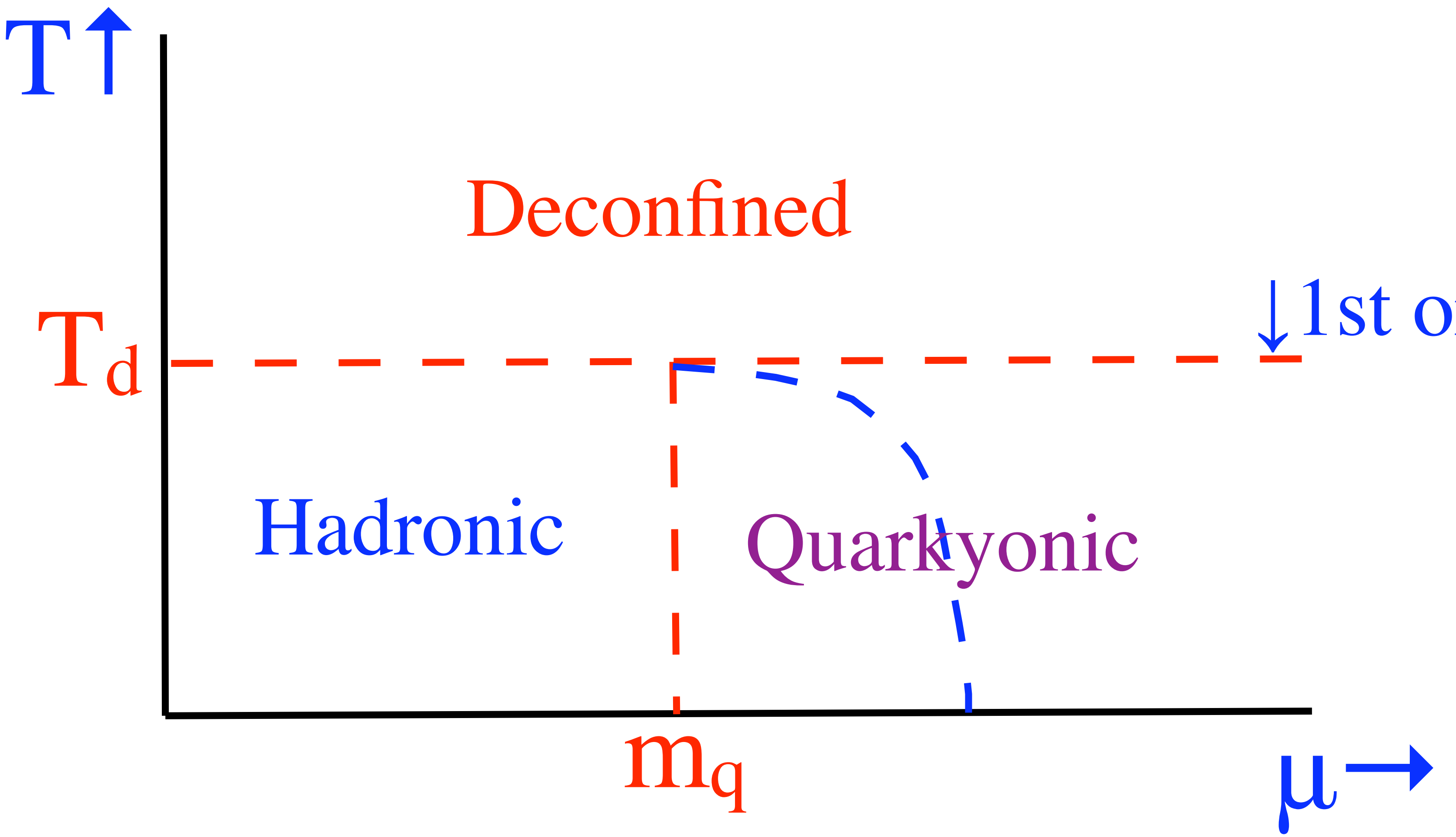
Chemical potential only matters when larger than mass:
$\mu_{\rm Baryon} > M_{\rm Baryon}$. Define $m_{\rm quark} = M_{\rm Baryon}/N_c$; so $\mu > m_{\rm quark}$.

"Box" for $T < T_c$; $\mu < m_{\rm quark}$: confined phase baryon free, since their mass ~ N_c
Thermal excitation ~ $\exp(-m_B/T) \sim \exp(-N_c) = 0$ at large N_c.
So hadronic phase in "box" = mesons & glueballs only, *no* baryons.

Phase diagram at $N_c = \infty$, I

At *least* three phases. At large N_c, can use pressure, P, as order parameter.
Hadronic (confined): $P \sim 1$. Deconfined, $P \sim N_c^2$. Thorn '81

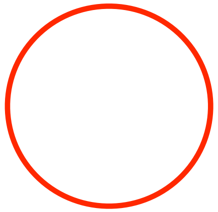
Quarks or baryons = "quark-yonic", $P \sim N_c$. Chiral symmetry restoration?
N.B.: mass threshold at m_q neglects (possible) nuclear binding, Son



Nuclear matter at large N_c

$\mu_{Baryon} = \sqrt{k_F^2 + M^2}$, k_F = Fermi momentum of baryons.
Pressure of ideal baryons density times energy of non-relativistic baryons:

$$P_{\text{ideal baryons}} \sim n(k_F)\,\frac{k_F^2}{M} \sim \frac{1}{N_c}\,\frac{k_F^5}{\Lambda_{QCD}}$$

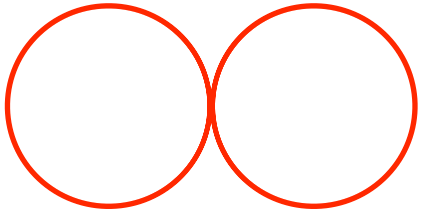


This is small, ~ $1/N_c$. The pressure of the I = J tower of resonances is as small:

$$\delta P_{\text{resonances}} \sim \frac{1}{M}\,\frac{k_F^8}{\Lambda_{QCD}^3} \sim \frac{1}{N_c}\,\frac{k_F^8}{\Lambda_{QCD}^4}$$

Two body interactions are huge, ~ N_c in pressure.

$$\delta P_{\text{two body int.'s}} \sim N_c\,\frac{n(k_F)^2}{\Lambda_{QCD}^2} \sim N_c\,\frac{k_F^6}{\Lambda_{QCD}^2}$$



At large N_c , nuclear matter is dominated by potential, not kinetic terms!
Two body, three body... interactions *all* contribute ~ N_c .

Window of nuclear matter

Balancing $P_{\text{ideal baryons}} \sim P_{\text{two body int.'s}}$, interactions important very quickly,

$$k_F \sim \frac{1}{N_c^2}\,\Lambda_{QCD}$$

For such momenta, only two body interactions contribute.

By the time $k_F \sim 1$, *all* interactions terms contribute $\sim N_c$ to the pressure.

But this is *very* close to the mass threshold,

$$\mu - m_q = \frac{\mu_B - M}{N_c} = \frac{k_F^2}{2MN_c} \sim \frac{1}{N_c^2}\,k_F^2$$

Hence "ordinary" nuclear matter is only in a *very* narrow window.

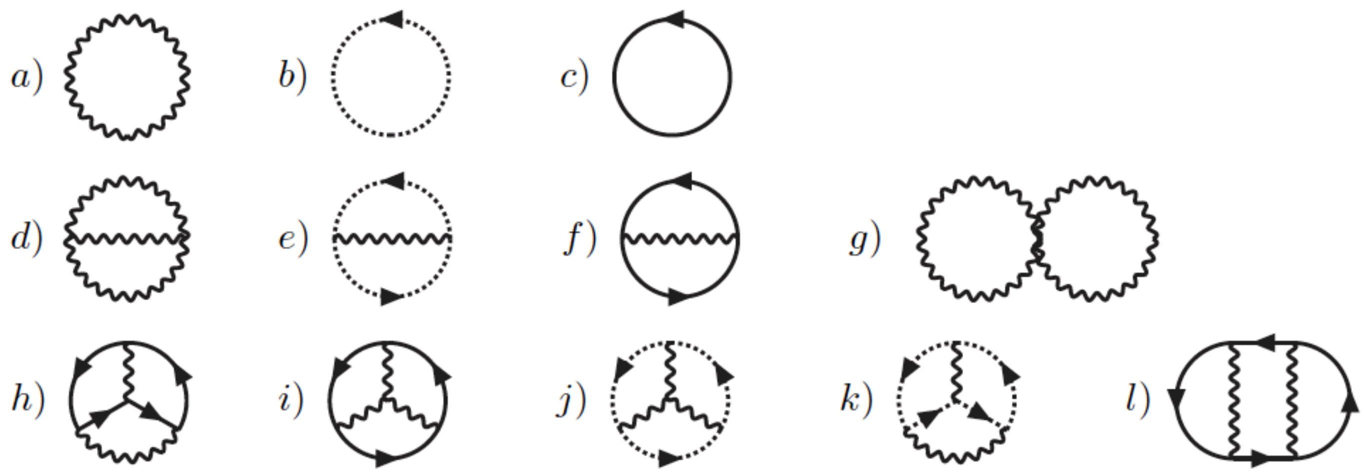
One quickly goes to a phase with pressure $P \sim N_c$.

So is it baryons, or quarks?

Perturbative pressure

At high density, $\mu >> \Lambda_{QCD}$, compute $P(\mu)$ in QCD perturbation theory.

To $\sim g^4$, Freedman & McLerran ('77)[3]; Ipp, Kajantie, Rebhan, & Vuorinen '06



At $\mu \neq 0$, only diagrams with at least one quark loop contribute. Still...

$$P_{\mathrm{pert.}}(\mu) \sim N_c N_f \; \mu^4 \; F_0(g^2(\mu/\Lambda_{QCD}), N_f)$$

For $\mu >> \Lambda_{QCD}$, but $\mu \sim N_c^0 \sim 1$, calculation reliable.

Compute $P(\mu)$ to $\sim g^6$, g^8... ? No "magnetic mass" at $\mu \neq 0$, well defined $\forall$ $(g^2)^n$.

"Quarkyonic" phase at large N_c

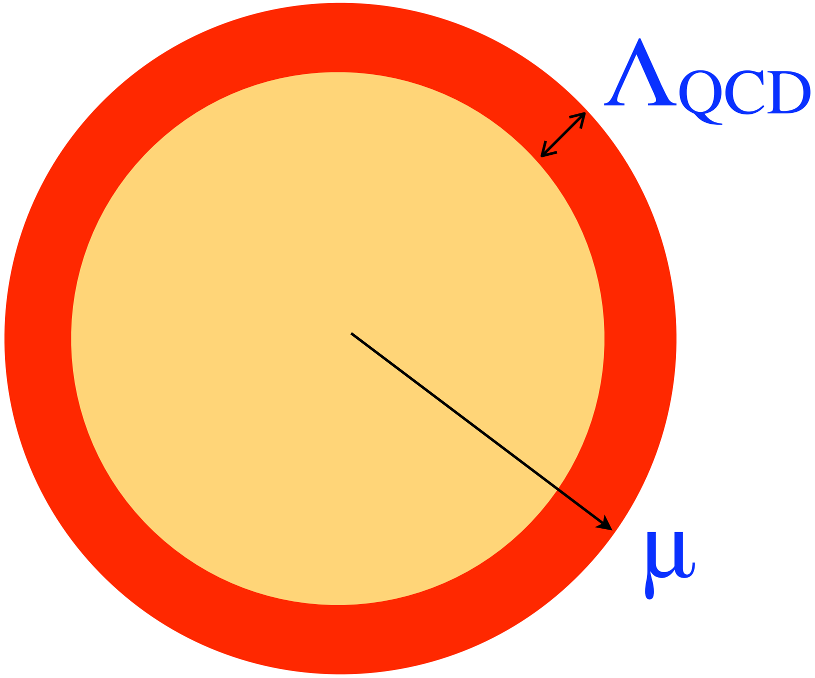
As gluons blind to quarks at large N_c, for $\mu \sim N_c^0 \sim 1$, *confined* phase for $T < T_d$

This includes $\mu >> \Lambda_{QCD}$! Central puzzle. We suggest:

To left: Fermi sea.

Deep in the Fermi sea, $k << \mu$,
looks like quarks.

But: within $\sim \Lambda_{QCD}$ of the Fermi surface,
confinement => *baryons*



We term combination "quark-yonic"

OK for $\mu >> \Lambda_{QCD}$. When $\mu \sim \Lambda_{QCD}$, baryonic "skin" entire Fermi sea.

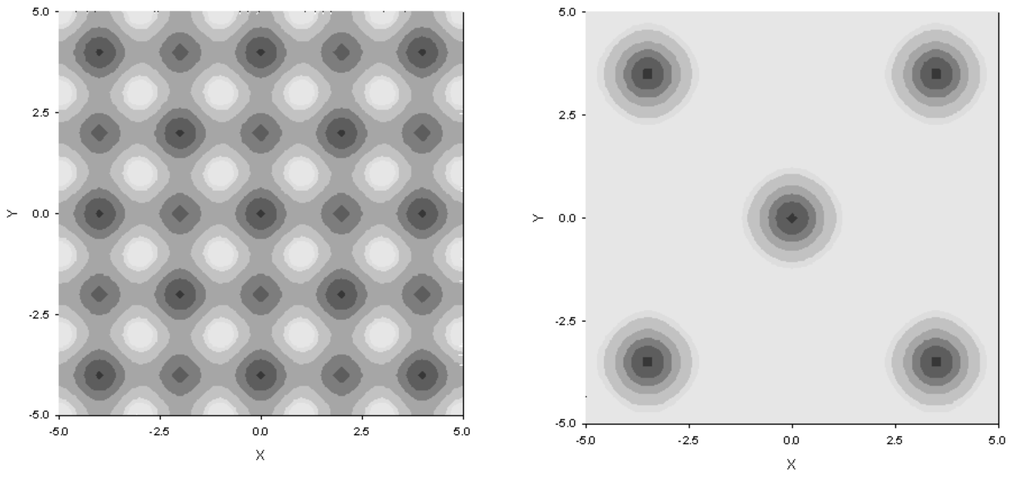
But what about chiral symmetry breaking?

Skyrmion crystals

Skyrmion crystal: soliton periodic in space.

Kutschera, Pethick & Ravenhall (KPR) '84; Klebanov '85 + ...

Lee et al , hep-ph/0302019 =>



At low density, chiral symmetry broken by Skyrme crystal, as in vacuum.

Chiral symmetry *restored* at nonzero density: $< U > = 0$ in each cell.

Goldhaber & Manton '87: due to "half" Skyrmion symmetry in each cell.

Forkel, Jackson et al, '89: excitations *are* chirally symmetric.

Easiest to understand with "spherical" crystal, KPR '84, Manton '87.

Take same boundary conditions as a single baryon, but for sphere of radius R:

At $r = R$: $\pi^a = 0$. At $r = 0$, $\pi^a = \pi\, r^a/r$. Density one baryon/($4\,\pi\, R^3/3$).

At high density, term $\sim \kappa$ dominates, so energy density $\sim$ baryon density$^{4/3}$.

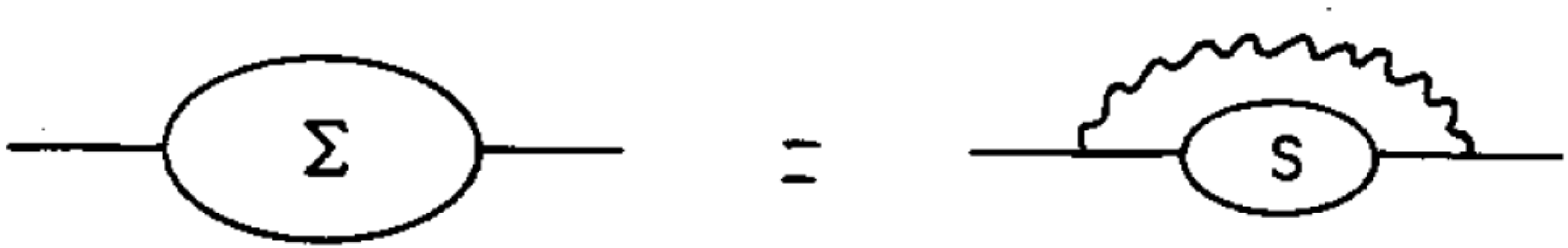
Like perturbative QCD! Accident of simplest Skyrme Lagrangian.

Schwinger-Dyson equations at large N_c: 1+1 dim.'s

't Hooft '74: as gluons blind to quarks at large Nc, S-D eqs. simple for quark:

Gluon propagator, and gluon quark anti-quark vertex unchanged.

To leading order in $1/N_c$, only quark propagator changes:



't Hooft '74: in 1+1 dimensions, single gluon exchange generates linear potential,

$$g_{2D}^2 \int dk \, \frac{e^{ikr}}{k^2} \sim g_{2D}^2 \, r$$

In vacuum, Regge trajectories of confined mesons. Baryons?

Solution at $\mu \neq 0$? Should be possible, not yet solved.

Thies et al '00...06: Gross-Neveu model has crystalline structure at $\mu \neq 0$

Schwinger-Dyson eqs. at large N_c: 3+1 dim.'s

Glozman & Wagenbrunn 0709.3080: in 3+1 dimensions, confining gluon propagator, $1/(k^2)^2$ as $k^2 \to 0$:

$$g^2 \int d^3k \, \frac{e^{ikr}}{k^2} \left(1 + \frac{\sigma}{k^2}\right) \sim g^2 \, \sigma \, r \; , \; r \to \infty$$

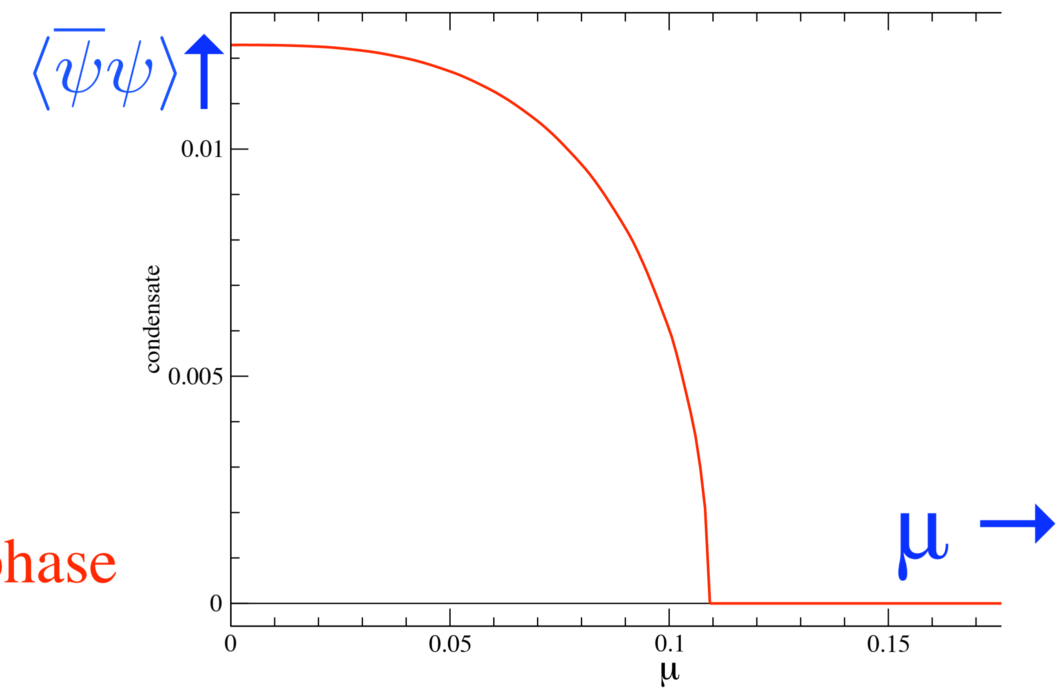
Involves mass parameter, σ. At $\mu = 0$, $\langle \overline{\psi}\psi \rangle = (.23\sqrt{\sigma})^3$

Take S-D eq. at large Nc, so confinement unchanged by $\mu \neq 0$.

Find chiral symmetry restoration at

$$\mu_\chi = .11\sqrt{\sigma}$$

Hence: in two models at $\mu \neq 0$, chiral symmetry restoration in *confined* phase



Asymptotically large μ

For $\mu \sim (N_c)^p$, $p > 0$, gluons no longer blind to quarks. Perturbatively,

$$P_{\text{pert.}}(\mu, T) \sim N_c N_f \, \mu^4 \, F_0 \; , \; N_c N_f \, \mu^2 \, T^2 \, F_1 \; , \; N_c^2 \, T^4 \, F_2 \; .$$

First two terms from quarks & gluons, last only from gluons. Two regimes:

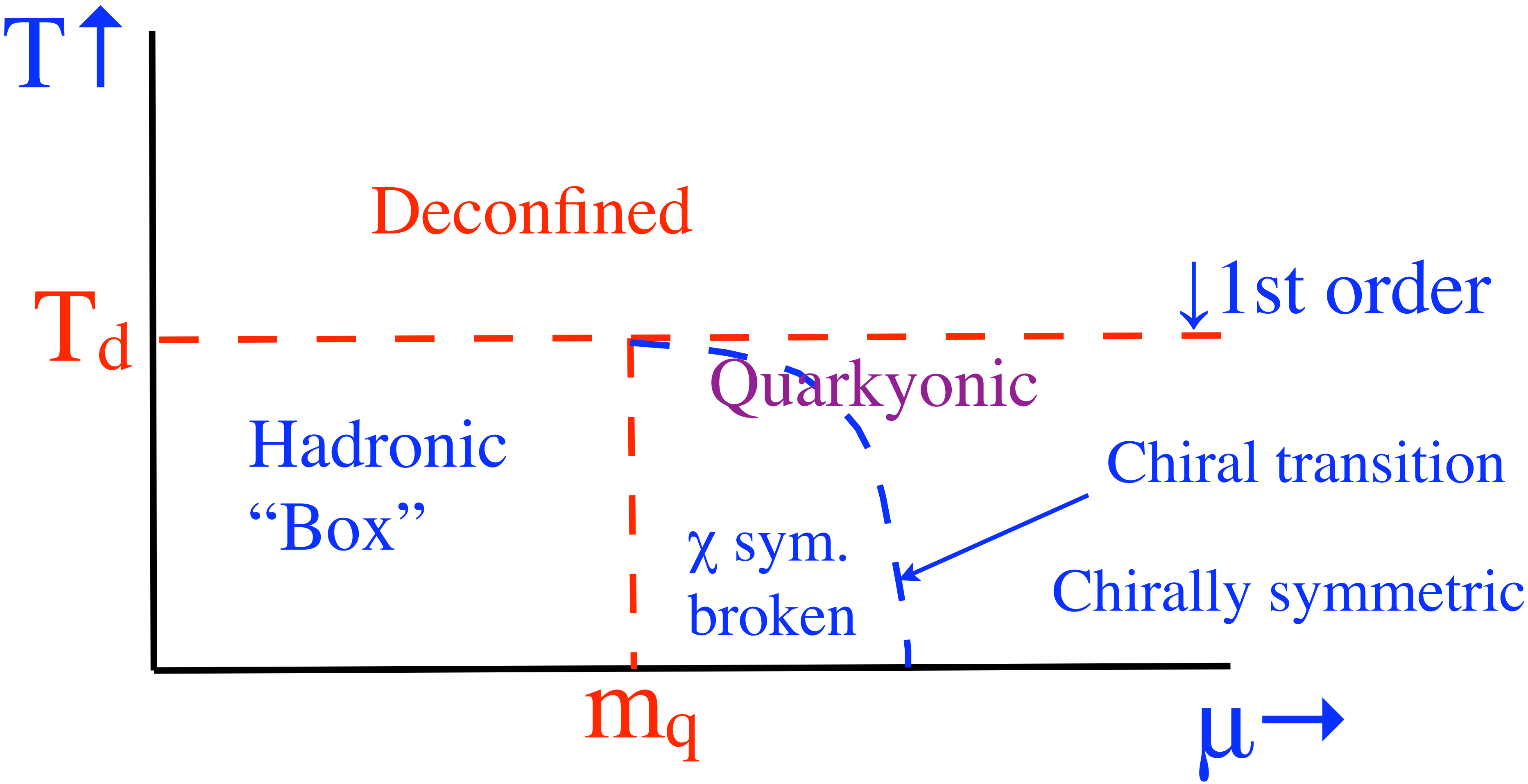
$\mu \sim N_c^{1/4} \Lambda_{QCD}$: $N_c \mu^4 F_0 \sim N_c^2 F_2 \sim N_c^2 >> N_c \mu^2 F_1 \sim N_c^{3/2}$.
Gluons & quarks contribute equally to pressure; quark cont. T-independent.

$\mu \sim N_c^{1/2} \Lambda_{QCD}$: New regime: $m^2_{Debye} \sim g^2 \mu^2 \sim 1$, so gluons feel quarks.

$N_c \mu^4 F_0 \sim N_c^3 >> N_c \mu^2 F_1 , N_c^2 F_2 \sim N_c^2$.
Quarks dominate pressure, T-independent.

Eventually, first order deconfining transition can either:
end in a critical point, or bend over to $T = 0$: ?

Phase diagram at $N_c = \infty$, II



We suggest: quarkyonic phase includes chiral trans. Order by usual arguments.

Mocsy, Sannino & Tuominen '03: splitting of transitions in effective models

But: quarkyonic phase confined. Chirally symmetric baryons?

Chirally symmetric baryons

B. Lee, ‘72; DeTar & Kunihiro ’89; Jido, Oka & Hosaka, hep-ph/0110005; Zschiesche et al nucl-th/0608044. Consider *two* baryon multiplets. One usual nucleon, other parity partner, transforming *opposite* under chiral transformations:

$$\psi_{L,R} \rightarrow U_{L,R}\, \psi_{L,R} \;\; ; \;\; \chi_{L,R} \rightarrow U_{R,L}\, \chi_{L,R}$$

With two multiplets, can form chirally symmetric (parity even) mass term:

$$\psi_L\, \chi_R \, - \, \psi_R\, \chi_L \, + \, \chi_R\, \psi_L \, - \, \chi_L\, \psi_R$$

Also: usual sigma field, $\Phi \rightarrow U_L\, \Phi\, U_R^\dagger$, couplings for linear sigma model:

$$g_1\, \psi_L\, \Phi\, \psi_R \, + \, g_2\, \chi_R\, \Phi\, \chi_L$$

Generalized model at $\mu \neq 0$: D. Fernandez-Fraile & RDP ’07...

Anomalies?

't Hooft, '80: anomalies rule *out* massive, parity doubled baryons in vacuum:
No massless modes to saturate anomaly condition

Itoyama & Mueller'83; RDP, Trueman & Tytgat '97:
At $T \neq 0$, $\mu \neq 0$, anomaly constraints *far* less restrictive (many more amplitudes)
E.g.: anomaly unchanged at $T \neq 0$, $\mu \neq 0$, but Sutherland-Veltman theorem *fails*

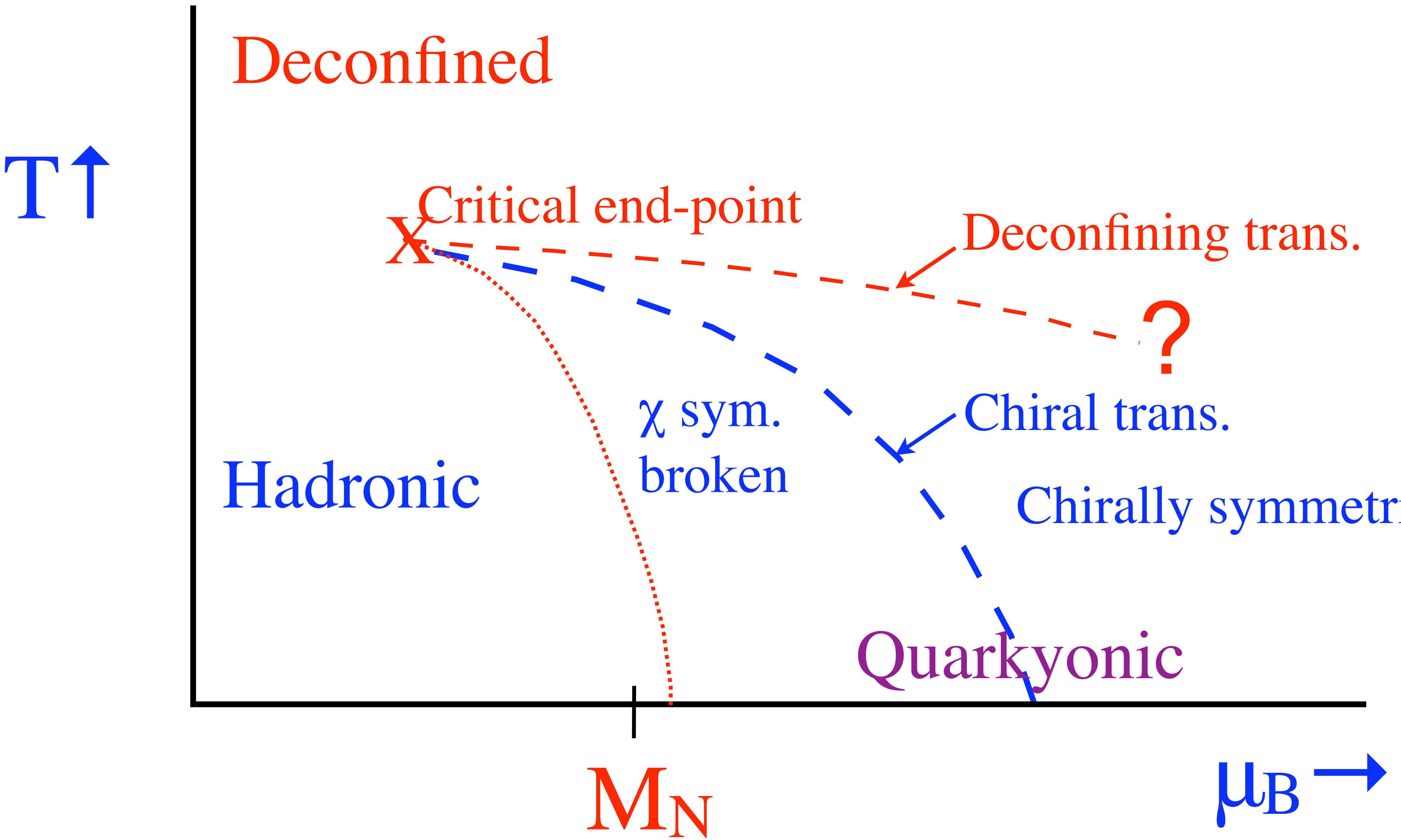
Must do: show parity doubled baryons consistent with anomalies at $\mu \neq 0$.
At $T \neq 0$, $\mu = 0$, no massless modes. Anomalies probably rule out model(s).
But at $\mu \neq 0$, *always* have massless modes near the Fermi surface.

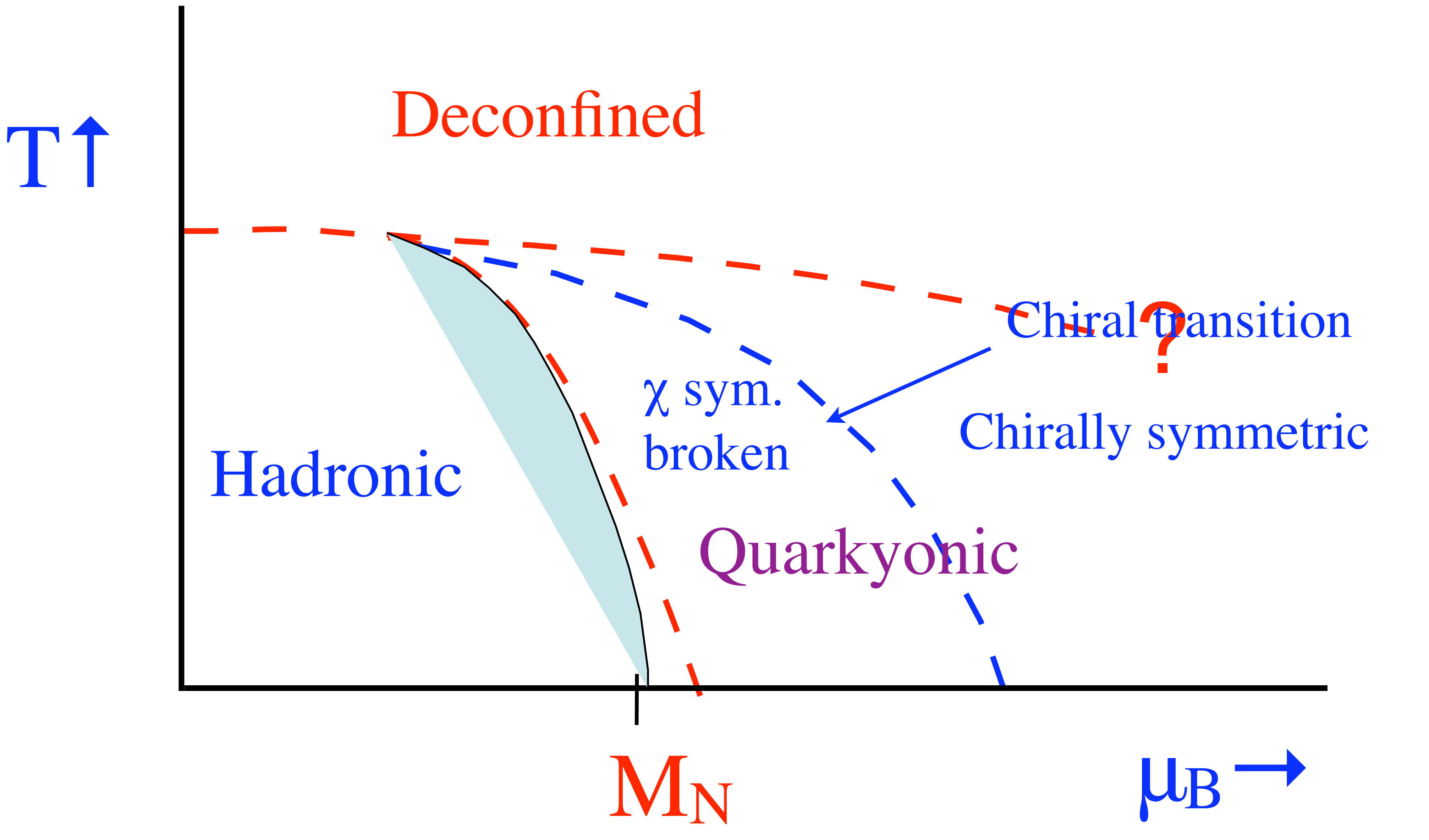
Casher '79: heuristically, confinement => chiral sym. breaking in vacuum
Especially at large N_c, carries over to $T \neq 0$, $\mu = 0$.
Does *not* apply at $\mu \neq 0$: baryons strongly interacting at large N_c.

Banks & Casher '80: chiral sym. breaking from eigenvalue density at origin.
Splittorff & Verbaarschot '07: at $\mu \neq 0$, eigenvalues spread in complex plane.
(Another) heuristic argument for chiral sym. restoration in quarkyonic phase.

Guess for phase diagram in QCD

Pure guesswork: deconfining & chiral transitions split apart at critical end-point?
Line for deconfining transition first order to the right of the critical end-point?
Critical end-point for deconfinement, or continues down to T=0?





T
Deconfined
Chiral transition
?
χ sym.
broken
Chirally symmetric
Hadronic
Quarkyonic
M_N
μ_B